A non-convex meta-frontier Malmquist index for measuring productivity over time

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1. Introduction

Caves et al. (1982) were the first who suggested that the Malmquist index – proposed initially for consumption analysis – can be employed in the context of productivity measurement. It then took 10 years until Färe et al. (1992) adapted the work of Caves et al. (1982) in order to apply data envelopment analysis (DEA) models for measuring productivity changes over time. In the same paper, Färe et al. (1992) also showed how the DEA-based Malmquist index can be exhibited as the product of the technical change and efficiency change components as two important drivers of productivity. However, since the introduction of the index, a few limitations have been faced by researchers. As this form of the Malmquist index uses the geometric mean of two measures of productivity change – which refer to the adjacent time periods under consideration – it fails circularity. Infeasibilities can also occur when DEA models under variable returns to scale (VRS) are used to compute and decompose the index. Over the last two decades, the depicted shortcomings have motivated researchers to focus on the methodological development of the Malmquist index and its decomposition. A thorough review of the family of the Malmquist indices can be found in Afsharian and Ahn (2015).

Among the different frameworks of the Malmquist index, the meta-frontier Malmquist index has recently begun to receive considerable attention by researchers. The reason is that not only this form of the Malmquist index can overcome the above-mentioned issues but also offers a number of other interesting features. This index was proposed first by Hayami and Ruttan (1970) and developed further by Battese et al. (2004) in the area of a parametric productivity analysis for the estimation of technical efficiencies and technology gaps for observations that may not have the same technology. Their approach assumes that – within the same industry – there are several well-defined groups of observations, which operate under their own local technologies. Accordingly, local frontiers are constructed by considering all observations belonging to the same group while the meta-frontier is the envelope of the group frontiers. The primal version of this index no longer measures the productivity change between two time periods, but provides a cross-sectional comparison of the performance of groups of DMUs in a static setting. Therefore, it has recently been enhanced by Pastor and Lovell (2005) and Oh and Lee (2010) as a tool to also measure productivity change over time (see also Portela and Thanassoulis 2008; Oh 2010; Afsharian and Ahn 2015).

According to the design of the meta-frontier Malmquist index, a single meta-technology is constructed from data for observations belonging to all groups and observed in all time periods. This meta-technology then serves as a "global" benchmark, representing the best experienced

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technology among all groups and over all time periods in the analysis. On this basis, one not only can measure the within-group efficiency of units at a specific period of time but also capture how their efficiency has changed with regard to the meta-technology. Although the central concept of this index is compelling, it suffers from a drawback: the meta-technology is formed by the convex union of all experienced group technologies over time. Taking into account even a static setting (where only a cross-sectional analysis is applied), researchers argue that any meta-technology, which is formed as the union of "even convex group technologies", is unlikely to be convex (see, e.g., Huang et al. 2013; Kerstens et al. 2015). This can be more problematic when the index also includes a time component to measure productivity change over a number of time periods. In this case, the convexication neglects that the technology under which each group of units operates can change over time. This negligence may lead to an incorrect estimation of the meta-frontier such that the corresponding results of productivity will not properly reflect the performance.

Against this background, we propose a new way of estimating the meta-technology, which applies the minimum extrapolation principle on the aggregation of the experienced group technologies over time. As will be shown, the resulting index, called the non-convex meta-frontier Malmquist index, provides more accurate results compared to the existing meta-frontier Malmquist index. The new index also preserves the role of each group technology – observed at the specific time period – in the estimation of the meta-technology. Therefore, individual characteristics of the group technologies can later be traced in measuring productivity change. In particular, this includes information about group technologies which contribute significantly to the shape of the meta-technology over time. This unique feature of the suggested approach plays a crucial role in measuring and analyzing productivity, where a further diagnosis of individual performances is required. With respect to both computational and test properties, the proposed index also possesses the circularity property, generates a single measure of productivity change and is immune to infeasibility under VRS. Similar to traditional indices, it can be decomposed into the standard components such as efficiency change and best practice change.

The rest of the paper unfolds as follows: after some preliminaries and technical background in Section 2.1, we depict our idea of estimating the meta-frontier technology in Section 2.2. In Section 2.3, the proposed meta-frontier Malmquist productivity index is formulated mathematically. Section 3 illustrates the new index and its properties by means of an empirical application to KONE corporation. The paper concludes with a summary and an outlook on future research opportunities in Section 4.

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2. The new meta-frontier Malmquist index

2.1 Preliminaries and technical background

Suppose that there exists a panel of n DMUs which can be partitioned into G (G>1) distinct groups observed in T time periods. Let each group g (g=1,…,G) include δ_g DMUs $(X_j^{g,t}, Y_j^{g,t}) \in \Re_+^m \in \Re_+^s$ (j=1,..., δ_g), where $X_j^{g,t} = (x_{1j}^{g,t}, x_{2j}^{g,t}, ..., x_{mj}^{g,t})$ and $Y_j^{g,t} = (y_{1j}^{g,t}, y_{2j}^{g,t}, ..., y_{sj}^{g,t})$ are non-negative and non-zero vectors of inputs and outputs, respectively, observed in period t (t=1,…,T). Following O'Donnell et al. (2008), we assume that all DMUs in each group g operate under the same technology, resulting from, e.g., the same resource, regulatory or other environmental constraints. Hence, each local technology of group g in time period t can be represented by a production possibility set (PPS) or technology set (in the following also abbreviated as "technology") of feasible input-output combinations as follows:

$$PPS^{g,t} = \left\{ (X^{g,t}, Y^{g,t}) \in \Re_+^m \times \Re_+^s \,\middle|\, X^{g,t} \text{ can produce } Y^{g,t} \right\} . \tag{1}$$

Throughout the paper, without loss of generality, we assume that the local technologies in (1) satisfy non-emptiness, free disposability, convexity and minimum extrapolation. The following analysis may be straightforwardly extended to other types of technologies with other axioms. Taking into account these axioms, the local technologies in (1) can be expressed precisely by means of the following technology sets:

$$PPS^{g,t} = \left\{ \begin{array}{l} (X^{g,t}, Y^{g,t}) \in \Re_+^m \times \Re_+^s \,\middle|\, x_i^{g,t} \geq \sum_{j=1}^{\delta_g} \lambda_j^{g,t} x_{ij}^{g,t},\ y_r^{g,t} \leq \sum_{j=1}^{\delta_g} \lambda_j^{g,t} y_{rj}^{g,t}, \\ \sum_{j=1}^{\delta_g} \lambda_j^{g,t} = 1, \quad \lambda_j^{g,t} \geq 0, \quad j = 1, ..., \delta_g \end{array} \right\}. \tag{2}$$

With respect to the definition of $PPS^{g,t}$ in (2), one can measure the efficiency of a DMU against the frontier of a particular group g (g=1,…,G) at the specific point of time t (t=1,…,T). Moreover, we may also measure a "within-period" efficiency by means of a contemporaneous benchmark technology as (see, e.g., O'Donnell et al. 2008; Huang et al. 2013)

$$PPS^{M,t} = PPS^{1,t} \cup PPS^{2,t} \cup ,..., \cup PPS^{G,t} \tag{3}$$

where $PPS^{M,t}$ is formed by the aggregation of all group technologies in time period t (t=1,…,T). It should be noted that using (3) as a benchmark in a specific period t provides a cross-sectional comparison of the performance of groups of DMUs in a static setting, i.e., the measurement is done in a specific time period t. In order to make the comparison dynamic, (3) has to be modified. This can be done by the concept of the meta-frontier Malmquist index as follows:

Based on, e.g., Oh and Lee (2010), the meta-frontier Malmquist index for a DMU_p (the unit under evaluation) which belongs to group g (g=1,…,G), regarding two time periods t and t+1, is defined as:

$$MI^M(X_p^{g,t+1}, Y_p^{g,t+1}, X_p^{g,t}, Y_p^{g,t}) = \frac{Eff^M(X_p^{g,t+1}, Y_p^{g,t+1})}{Eff^M(X_p^{g,t}, Y_p^{g,t})} . \tag{4}$$

In this formula, $Eff^M(X_p^{g,t}, Y_p^{g,t})$ and $Eff^M(X_p^{g,t+1}, Y_p^{g,t+1})$ represent the two required input-oriented meta-efficiencies, which can be determined as:

$$Eff^M(X_p^{g,k}, Y_p^{g,k}) = \min\left\{ \theta_p^k : (\theta_p^k X_p^{g,k}, Y_p^{g,k}) \in PPS^M \right\}, \quad k = t, t+1. \tag{5}$$

PPS^M is the meta-technology, which aggregates all group technologies over all time periods as

$$PPS^M = \bigcup_{t=1}^{T} \bigcup_{g=1}^{G} PPS^{g,t} . \tag{6}$$

More details about the meta-frontier Malmquist index and its potentials to provide interesting insights into DEA applications can be found in a series of papers such as in Portela and Thanassoulis (2008), Oh (2010), Chen and Yang (2011), Portela et al. (2011), Afsharian and Ahn (2015), Choi et al. (2015) as well as Kerstens et al. (2015).

2.2 Motivation and graphical explanations

In the existing form of the meta-frontier Malmquist index, all observations from all groups in all periods are assumed to be theoretically and potentially able to access a single best practice technology. This meta-technology – which is assumed to be available to the whole industry in

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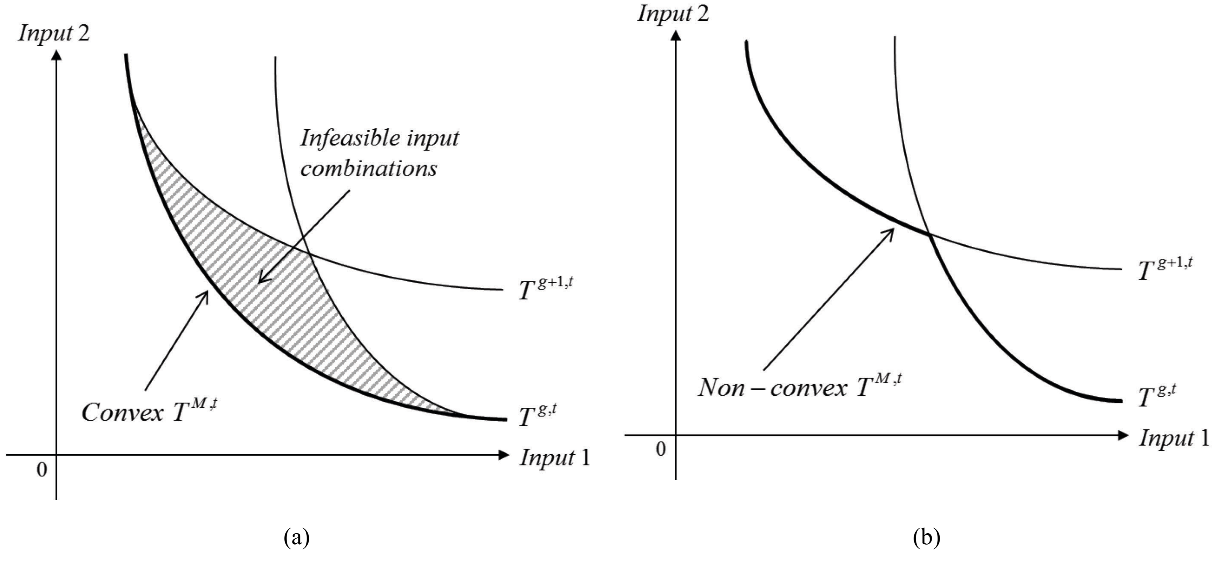
which the DMUs operate – is then obtained by the "convex aggregation" of the group technologies over time (see, e.g., Oh and Lee 2010; Chen and Yang 2011). On this basis, all observations from different groups and time periods are accepted to form the meta-benchmark technology. This means that the characteristics of the group technologies are implicitly assumed to remain unaltered over time, i.e., it is assumed to be no technical differences between different groups of DMUs which are observed over time. This is clearly inconsistent with the primary setting of the problem by which the DMUs are partitioned to G distinct groups observed in T time periods. As a consequence, although observations in each time period can be considered to be acceptable to form the respective group technology set in a specific time period, including all observations from all groups in all periods in the analysis (to estimate the meta-technology) is questionable, as illustrated also graphically in the following.

Let us suppose that there exist two group technologies g and g+1 in a single time period t. We also assume that there are two inputs and a single output, and the observations have the same level of output. The corresponding local technologies $T^{g,t}$ and $T^{g+1,t}$ are depicted in Fig. 1. According to the definition in (3), one can form a respective contemporaneous benchmark technology to provide a cross-sectional comparison of the performance of these two groups of DMUs in the single time period t. Following the existing method of aggregation, the resulting contemporaneous technology – indicated by $Convex\ T^{C,t}$ – will be the one shown in Fig. 1 (a). Considering the frontier of this technology set, we can see that there are areas – shown by "infeasible input combinations" – which have neither been experienced nor producible in reality. In fact, this area is only formed as a direct consequence of the imposed convexity assumption between these groups to easily estimate the contemporaneous technology $Convex\ T^{C,t}$.

Figure 1. Convex and non-convex estimations of a contemporaneous technology set



One can now compare this result to the more precise aggregation of the group technologies as shown in Fig. 1 (b) by $Non-convex\ T^{C,t}$. As can be seen here, the aggregation of the distinct group technologies $T^{g,t}$ and $T^{g+1,t}$ applies the minimum extrapolation principle. Hence, the resulting $Non-convex\ T^{C,t}$ provides a pure union of what has really been occurred rather than also having an additional area resulting from the convexication between these groups.

The above graphical example shows that assuming convexity even between observations originating from different group technologies is a strong premise, while in a multi-period analysis this phenomenon becomes more problematic. The reason is that not only the business environment but also, e.g., government rules or regulations, policy directives and economic conditions, under which the DMUs operate, can change significantly over time. Therefore, convex combinations of units belonging to different time periods may also once more reduce the accuracy of the estimation of the meta-technology. A graphical example of this can be seen in Fig. 2, which depicts a meta-technology formed from the two group technologies *g* and *g*+1 over two time periods *t* and *t*+1.

As can be seen in Fig. 2 (a), the existing meta-frontier Malmquist index proposes a meta-technology which is the convex envelope of all group technologies over time, i.e., the convex aggregation of $T^{g,t}$, $T^{g+1,t}$, $T^{g,t+1}$ and $T^{g+1,t+1}$, indicated by $Convex\ T^{M}$. This result can be compared to our proposed estimation of the meta-technology $Non-convex\ T^{M}$ shown in Fig. 2 (b). A comparison between these two forms highlights how the convexication can produce virtual points which are the result of a poor estimation of the meta-technology, marked by the shaded area in Fig. 2 (a). It should be noted that even if contemporaneous technology sets (i.e., $T^{C,t}$ and $T^{C,t+1}$) were to satisfy convexity (e.g., perhaps in order to have a simple approximation), as we expect that the environment can change over time, there would be no reason why the union of these contemporaneous technology sets should be convex to estimate the meta-technology. Hence, we conclude that the proposed $Non-convex\ T^{M}$ as the meta-technology is a more accurate and appropriate estimate of the best practice technology, which has really been experienced over time.

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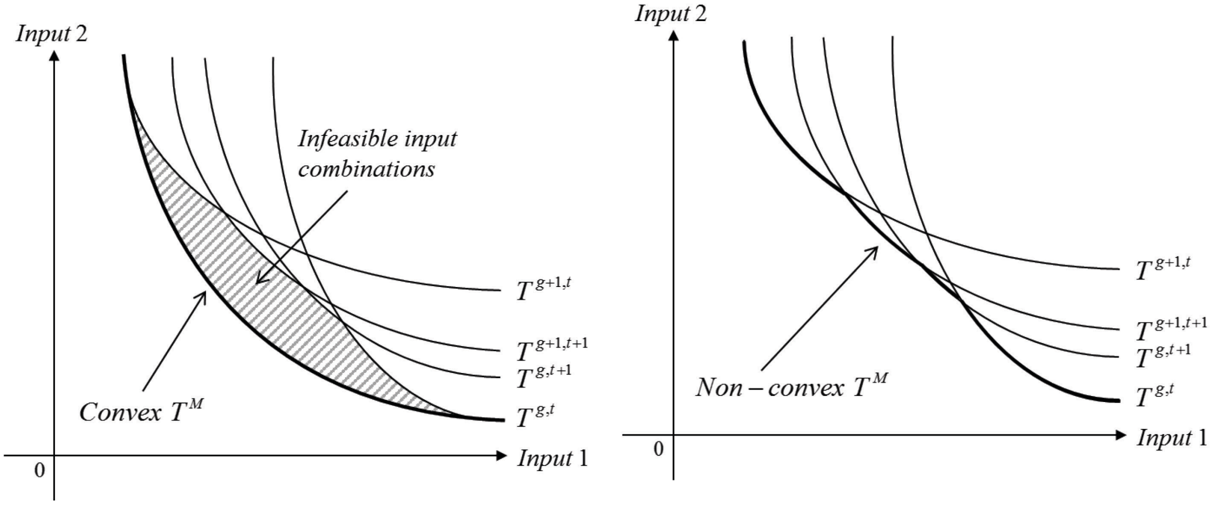
Figure 2. Convex and non-convex estimations of a meta-technology set



(a) (b)

In addition to the above remark about the estimation of the meta-technology, a closer look at the structure of $Convex\ T^M$ and $Non-convex\ T^M$ in Fig. 2 reveals another unique feature of the new form of the benchmark technology. A comparison between both forms shows that the proposed benchmark technology preserves the role of each group technology – observed at a specific time period – in the estimation of the meta-technology, i.e., information about local group technologies are not mixed. Unlike in $Convex\ T^M$, our approach allows tracing individual characteristics of the group technologies while measuring productivity change. In particular, information about group technologies, which contribute significantly to the shape of the meta-technology (in the following called "superior group technologies"), is revealed. This property of the $Non-convex\ T^M$ plays a crucial role in measuring and analyzing productivity, where a further diagnosis of individual performances is required.

2.3 The proposed meta-frontier Malmquist index framework

In this section, we formulate the meta-frontier Malmquist productivity index based on the proposed meta-technology. In accordance with the graphical example given in Section 2.2, this meta-technology is formed by the pure union of all local technologies in (3) as

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$$PPS_{NC}^{M} = \bigcup_{t=1}^{T}\bigcup_{g=1}^{G} PPS^{g,t} \tag{7}$$

where PPS_{NC}^{M} has been denoted by a subscript "NC" to emphasize that the meta-technology is now formed based on the non-convex union of the group technologies. This representation of the meta-technology can be precisely modeled by considering a number of mathematical axioms as follows:

1. (Non-emptiness). The observed $(X_j^{g,t}, Y_j^{g,t}) \in PPS_{NC}^{M}$, g=1,…,G; t=1,…,T; j=1,…, δ_g.
2. (Free disposability). If $(X,Y) \in PPS_{NC}^{M}$, $X' \geq X$, $Y' \leq Y$, then $(X',Y') \in PPS_{NC}^{M}$.
3. (Local convexity). If (X,Y) and $(\tilde{X},\tilde{Y}) \in PPS_{NC}^{M}$, then $\lambda(X,Y)+(1-\lambda)(\tilde{X},\tilde{Y}) \in PPS_{NC}^{M}$ for any $\lambda \in [0,1]$, provided that there exists t (t=1,…,T) and g (g=1,…,G) such that both (X,Y) and $(\tilde{X},\tilde{Y}) \in PPS^{g,t}$.
4. (Minimum extrapolation). PPS_{NC}^{M} is the smallest set which satisfies axioms 1-3.

With regard to the standard assumptions of DEA models, the meaning of axioms #1 and #2 is obvious. According to axiom #3, convex combinations among members of different group technologies are not required. Axiom #4 then ensures that PPS_{NC}^{M} will be the smallest set, which results from the pure union of the local group technologies. Taking into account these axioms, the definition of the meta-technology in (7) can mathematically be enhanced as:

$$PPS_{NC}^{M} = \bigcup_{t=1}^{T}\bigcup_{g=1}^{G}\left\{\begin{array}{l}(X^{g,t},Y^{g,t}) \in \Re_{+}^{m}\times\Re_{+}^{s} \mid x_i^{g,t} \geq \sum_{j=1}^{\delta_g}\lambda_j^{g,t}x_{ij}^{g,t},\ y_r^{g,t} \leq \sum_{j=1}^{\delta_g}\lambda_j^{g,t}y_{rj}^{g,t},\\ \sum_{j=1}^{\delta_g}\lambda_j^{g,t}=1, \qquad \lambda_j^{g,t} \geq 0, \qquad j=1,\ldots,\delta_g\end{array}\right\}. \tag{8}$$

On this basis, $Eff_{NC}^{M}(X_p^{g,t}, Y_p^{g,t})$, which captures the input-oriented efficiency of unit p belonging to a group g with the data from period t, can be determined as:

$$Eff_{NC}^{M}(X_p^{g,t}, Y_p^{g,t}) = \min\left\{\ \theta_{p(g,t)}^{M} : \left(\theta_{p(g,t)}^{M}X_p^{g,t}, Y_p^{g,t}\right) \in PPS_{NC}^{M}\ \right\}. \tag{9}$$

In order to compute the meta-efficiencies in (9), let us first assume that $Eff^{q,l}(X_p^{g,t}, Y_p^{g,t})$ (q=1,…,G and l=1,…,T) represent all local efficiencies of unit p (belonging to a group g with the data from period t) with the following definition:

$$Eff^{q,l}(X_p^{g,t},Y_p^{g,t}) = \min\left\{ \theta_{p(g,t)}^{q,l} : \left(\theta_{p(g,t)}^{q,l} X_p^{g,t}, Y_p^{g,t}\right) \in PPS^{q,l} \right\}, \;\; q=1,\dots,G; \;\; l=1,\dots,T. \tag{10}$$

According to (10), one can measure these local efficiencies of a unit p against any group technology q in time period l with the following linear programming problems:

$$Eff^{q,l}(X_p^{g,t},Y_p^{g,t}) = \min\left\{ \theta_{p(g,t)}^{q,l} \left| \begin{array}{ll} \sum_{j=1}^{\delta_g} \lambda_j^{g,t} x_{ij}^{g,t} \le x_{ip}^{q,l}\, \theta_{p(g,t)}^{q,l}, & i=1,\dots,m \\ \sum_{j=1}^{\delta_g} \lambda_j^{g,t} y_{rj}^{g,t} \ge y_{rp}^{q,l}, & r=1,\dots,s \\ \sum_{j=1}^{\delta_g} \lambda_j^{g,t} = 1 & \\ \lambda_j^{g,t} \ge 0, & \end{array} \right. \right\}. \tag{11}$$

Now with respect to the discrete nature of the meta-technology in (8), the meta-efficiencies in (9) can be computed by the following enumeration procedure:

$$Eff_{NC}^{M}(X_p^{g,t},Y_p^{g,t}) = \min_{\substack{l=1,\dots,T \\ q=1,\dots,G}} \left\{ Eff^{q,l}(X_p^{g,t},Y_p^{g,t}) \right\}. \tag{12}$$

In this procedure, determining $Eff_{NC}^{M}(X_p^{g,t},Y_p^{g,t})$ against the meta-technology is identical with finding the minimum value among $Eff^{q,l}(X_p^{g,t},Y_p^{g,t})$ for all l (l=1,…,T) and all q (q=1,…,G) in which $Eff^{q,l}(X_p^{g,t},Y_p^{g,t})$ can also be computed in advance by the corresponding DEA models in (11). It should be noted that as the DMU under evaluation is a real unit, at least one of its within-group efficiencies $Eff^{q,l}(X_p^{g,t},Y_p^{g,t})$ is feasible, e.g., it is enveloped by the technology in which it has been observed. According to (8), this guarantees that $Eff_{NC}^{M}(X_p^{g,t},Y_p^{g,t})$ is feasible. However, the above proposed formula (12) enumerates in its procedure all within-group efficiencies including those which might be infeasible for this unit. This can occur when DMU_p is not enveloped by the boundary of a particular group technology at a specific period of time. For overcoming this problem in the computation of (12), such infeasible results of efficiency can be replaced in advance by sufficiently big values.

According to the graphical examples in the previous section and also the way the meta-technology is formed by the convex and non-convex approaches, we have $Non-convex\ T^M \subseteq Convex\ T^M$. On this basis, $Eff^{M}(X_p^{g,t},Y_p^{g,t}) \le Eff_{NC}^{M}(X_p^{g,t},Y_p^{g,t})$ where

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$Eff^{M}(X_p^{g,t}, Y_p^{g,t})$ and $Eff_{NC}^{M}(X_p^{g,t}, Y_p^{g,t})$ denote the meta-efficiency of the convex and the non-convex approaches, respectively. This shows that a sufficient condition for the equality of these two approaches is the convexity of the meta-technology. Therefore, – as an extreme case from a theoretical point of view – if the grouping of units over time leads to a convex shape of the meta-technology, the efficiency results of these approaches will be exactly the same. However, in practical situations, the results tend to diverge when the meta-technology exhibits areas violating convexity in its shape.

Taking into account the definition of the best practice technology in (8), the proposed non-convex meta-frontier Malmquist index for DMU$_p$ which belongs to group g, regarding two time periods t and t+1, is defined as

$$MI_{NC}^{M}(X_p^{g,t+1}, Y_p^{g,t+1}, X_p^{g,t}, Y_p^{g,t}) = \frac{Eff_{NC}^{M}(X_p^{g,t+1}, Y_p^{g,t+1})}{Eff_{NC}^{M}(X_p^{g,t}, Y_p^{g,t})} \tag{13}$$

where $Eff_{NC}^{M}(X_p^{g,t}, Y_p^{g,t})$ and $Eff_{NC}^{M}(X_p^{g,t+1}, Y_p^{g,t+1})$ represent the meta-efficiencies which can be computed by the formula in (12). The proposed meta-frontier Malmquist index can also be represented by means of the following standard decomposition:

$$\begin{aligned} & MI_{NC}^{M}(X_p^{g,t+1}, Y_p^{g,t+1}, X_p^{g,t}, Y_p^{g,t}) = \\ & \underbrace{\frac{Eff^{g,t+1}(X_p^{g,t+1}, Y_p^{g,t+1})}{Eff^{g,t}(X_p^{g,t}, Y_p^{g,t})}}_{\text{Efficiency Change (EC)}} \times \underbrace{\left[\frac{Eff^{g,t}(X_p^{g,t}, Y_p^{g,t})}{Eff_{NC}^{M}(X_p^{g,t}, Y_p^{g,t})} \times \frac{Eff_{NC}^{M}(X_p^{g,t+1}, Y_p^{g,t+1})}{Eff^{g,t+1}(X_p^{g,t+1}, Y_p^{g,t+1})} \right]}_{\text{Best Practice Change (BPC)}} . \end{aligned} \tag{14}$$

The first component in (14) is the efficiency change (EC) component. It captures the change in the technical efficiency of the unit under evaluation between two time periods t and t+1. The second component is the best practice change (BPC) component, which indicates whether group technology g in time period t+1 in the region this unit operates is closer to or farther away from the meta-frontier than is group technology g in time period t. On this basis, if the value of the meta-frontier Malmquist index or any of its components is less than one, it denotes regress; a value greater than one implies progress, while a value of one indicates unchanged situation. We note here that since the meta-technology is obtained by the aggregation of the group technologies – similar to the original version of the meta-frontier Malmquist index –, the proposed index and its components are circular and immune to infeasibility under VRS.

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3. An application to KONE corporation

3.1 Background

In order to illustrate the proposed Malmquist index, we analyze a panel of 41 maintenance units of the company KONE (i.e. $n = 41$ DMUs) over the time period 2014-2015 (i.e. $t = 2014, 2015$). As an international engineering and service company headquartered in Helsinki, Finland, KONE corporation is recognized as one of the global leaders in the elevator and escalator industry (see KONE 2017b). Our numerical example focuses on "local maintenance units", which KONE is running in Germany. In order to oversee its maintenance units in an efficient way, KONE has partitioned them into four distinct managerial groups with regional headquarters in Hamburg, Berlin, Cologne and Munich, respectively (see KONE 2017a). For the sake of data anonymization, a randomly-selected number from 1-4 is given to each of these groups, to which we refer to, in the following, as G1, G2, G3 and G4.

The management of maintenance units in the four regions differ substantially according to differences in the business environment and resulting challenges faced by the groups. Therefore, each group operates independently in a self-reliant way in its business districts. This is done to make sure that they have enough flexibility to run the business according to demands of their local costumers. Each group is administrated by its own regional manager. The regional managers are responsible to enforce the overall company targets on the maintenance unit level. They ensure, e.g., that customized and high quality services are offered to local costumers. In order to achieve the centrally specified company objectives, the regional managers apply individual management concepts, customized strategies and local procedures which take into account different environmental constraints of their subordinated maintenance units.

In the context of our study, the representatives of KONE decided that two inputs and two outputs should be used to evaluate the maintenance units´ operational efficiency. The inputs are *the number of full-time equivalent employees (FTE)* and *weighted response time (WRT).* The first measures the number of employees in respect of total hours worked, whereby a FTE of 1.0 equals a full-time employee. The WRT adds up the time needed for maintenance tasks, repair tasks and the elimination of unexpected malfunctions with the relative weights of 0.2, 0.2 and 0.6, respectively. The two outputs are *the number of callouts (NOC)* and *total handling tasks (THT).* NOC is an indicator to measure the work quality of the maintenance units. It represents the number of registered customer orders and complains due to any interruptions of lift machines or a repair service with poor quality. Since both the customers and the company itself expect a

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minimum number of callouts, the NOC is considered as an undesirable output, which has to be minimized. The indicator THT, as the second output, presents the total number of installations to be maintained or repaired by the respective maintenance units. Since the company seeks to increase the number of commissioned tasks per unit (without further deteriorating the level of the other variables), this indicator is taken into account as an output to be maximized.

3.2 Model specification

With respect to the theory put forward in Section 2, assuming convexity between observations within four group technologies is a strong premise even when a cross-sectional analysis is done, while in a multi-period analysis, as in our case of KONE, this phenomenon becomes more problematic. In such a dynamic setting, the internal and external business environment, under which the four maintenance groups operate, can be expected to change over time.

A noteworthy example is the change in personnel positions in the top and regional management of KONE, which occurred during the course of 2014 (see KONE 2017c). Such changes often have a considerable effect on the strategy and policy directives of the whole company. It is therefore questionable if the managerial approaches used in 2014 are identical with those applied in the subsequent year. Moreover, high market dynamics over the period under study has caused a lift inventory growth of 2.64 %. Since the data set does not show any substantial changes in terms of the employed personnel, it can be deduced that the unit managers would have restructured their processes (e.g., optimization of the route planning) to handle their additional work load. Furthermore, a new law for the operation of lifts (i.e., the so-called industrial safety regulation) came into force in June 2015. The new law provided, among others, more frequent audits and also prescribed more restricted regulations regarding the maintenance of lifts (see KONE 2017d). This should have demanded more time to invest and thorough work of the technicians at the time.

In addition, we should also take into account some other changes in the external environment, such as changes in government rules or regulations, policy directives and economic conditions over the time period 2014-2015. These changes are also likely to make combinations of maintenance units from different time-periods unreasonable. As a consequence, including all convex combinations of all observations in all time periods in the analysis may result in a poor estimate of the meta-technology so that the corresponding results of productivity will not properly reflect the performance. In order to overcome this problem, the application of the proposed non-convex meta-frontier Malmquist index is suggested.

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The existing convex and the proposed non-convex meta-frontier Malmquist index have been formulated on the basis of the axioms already outlined in Sections 2.1 and 2.3, respectively. These axioms lead the DEA models to be under VRS assumption in both approaches. The reason is that if we, e.g., increase the inputs (i.e., FTE and WRT) by a certain factor, we cannot necessarily assume that the outputs (i.e., NOC and THT) will also increase by the same factor. The assumption of VRS also ensures that maintenance units are only benchmarked against units of a similar size. This is a property which was also demanded by the representatives of KONE to be satisfied in the analysis.

In order to deal with the undesirable output NOC, a linear transformation approach has been applied to its values. Accordingly, we multiply each value of this output by (-1) and find a proper translation amount to convert the negative data to non-negative data (for more details about this linear transformation see, e.g., Seiford and Zhu 2002). Furthermore, our analysis follows an input-oriented perspective as the maintenance groups are expected to minimize their inputs (i.e., FTE and WRT), controlling for their output levels (an overview of standard DEA models and their features can be found in, e.g., Thanassoulis 1997).

3.3 Results and managerial implications

The mathematical programming problems of the convex and non-convex meta-frontier Malmquist indices as well as their components have been encoded in AIMMS, version 3.13. Applied to our data set, Table 1 summarizes the results of the two frameworks. The first column represents the units and their corresponding group numbers. EC, BPC and MI refer to the efficiency change, the best practice change and the Malmquist index stemming from both approaches.

In order to have a general picture of the differences between the results of the two approaches in measuring productivity change, the Spearman's rank correlation test has been carried out. This non-parametric test determines the degree to which two numerical variables (e.g. MI) are monotonically related or associated (for more details about the Spearman test, see, e.g., Miah 2016). With respect to the results in Table 1, the Spearman's rank correlation coefficient concerning the MI is 0.905, meaning that there is a high congruity between the rankings in the two approaches. This is an interesting result as the proposed method here does not change entirely the concept and the structure of how the productivity change is measured. Nonetheless, the non-convex approach provides a more accurate set of results compared to the convex approach as will be investigated in the following.

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As can be seen in Table 1, the results of productivity change obtained by the two approaches diverge substantially for the majority of units. From 41 maintenance units in the data set, only twelve units (7, 8, 16, 17, 18, 20, 22, 23, 30, 34, 35 and 37) yield the same numerical values of the MI. For the other units, significant differences can be observed. Take unit #19 as an example: Its numerical value of the MI differs by around 7% – while our approach captures a positive change in productivity over time (i.e., 5.9%), the convex meta-frontier suggests a decline of 1.1%. From a theoretical point of view, this example gives interesting evidence of how a more accurate estimation of the meta-technology leads to a significant difference in the results. Taking a closer look at them, we have also checked which estimated values of productivity change represent properly the performance of unit #19 from a practical point of view. We have realized that the manager of this maintenance unit has undertaken various efforts (e.g., optimized route planning) to cope with an additional workload (i.e., a higher THT value) while improving its work quality (i.e., reducing its NOC value). Therefore, a productivity decline of 1.1% (as it is determined by the convex MI) is indeed counter-intuitive. In contrast, the productivity improvement of 5.9% attested by the non-convex MI corresponds closely to the practical expectations of KONE´s management.

As another example, the productivity change of unit #32 amounts to +11.5% with the convex MI, while there is no change shown by the non-convex MI. Analyzing our detailed results, we have observed that the convex approach includes different group technologies from both periods of time to measure the MI of this unit (i.e., unit #36 from period 1 and units #21 and #32 from period 2). This result (regardless of the value captured) is not readily acceptable by the management due to differences in group technologies over time (see Section 3.2 for a few examples of these changes). Furthermore, this combination has led to a very large value of the MI of +11.5%, which has also been recognized as a value far away beyond management's expectations of the performance of this unit. Unit #32 has the reputation of being one of the best performing maintenance units in the whole sample so that its performance is expected to be very high in both observed periods. However, only the non-convex approach has captured a full meta-efficiency of this unit in both periods, resulting in a MI of one. In contrast, the convex approach estimated a full meta-efficiency in the second period, but a meta-inefficiency in the first period, leading to a large unexpected positive change in productivity over time. This substantial productivity improvement of +11.5% has been considered as unrealistic by KONE´s management.

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The results in Table 1 also show that the EC component of the Malmquist index is identical for all units determined by the two approaches. The reason is that both the convex and non-convex meta-frontier Malmquist index apply the same ratio of efficiencies to capture the EC component (see formula 14 in Section 2.3). This leads to the conclusion that the discrepancies between the results of the MI originate exclusively from the different estimations of the meta-technology required for the computation of the respective BPC component. As Table 1 reports, the BPCs of some units in the existing framework are less than those in our proposed approach, while the opposite is true for some other units. As a key factor affecting the results of productivity change, the BPC component indicates a productivity loss or gain for the majority of maintenance units. Take again unit #19 as an example. As can be seen, a poor estimation of the technology and the corresponding result of the BPC suggest that the productivity of this unit has declined within the existing approach of the Malmquist index, while the enhanced method of estimation within our approach identifies an opposite direction. This underlines the serious drawback of the convex meta-frontier Malmquist index concerning the estimation of the benchmark technology and the resulting productivity values, which may lead obviously to wrong conclusions and policy recommendations.

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Table 1. Results obtained by the existing convex and the proposed non-convex meta-frontier Malmquist index

	Convex meta-frontier Malmquist index			*Non-convex meta-frontier Malmquist index*		
	EC	**BPC**	**MI**	**EC**	**BPC**	**MI**
Unit 1 (G1)	1.004	0.965	0.969	1.004	0.958	0.961
Unit 2 (G1)	1.087	0.971	1.056	1.087	0.968	1.052
Unit 3 (G1)	1.000	1.217	1.217	1.000	1.266	1.266
Unit 4 (G1)	1.000	1.178	1.178	1.000	1.076	1.076
Unit 5 (G1)	1.200	1.152	1.382	1.200	1.230	1.476
Unit 6 (G1)	1.049	1.079	1.131	1.049	1.100	1.154
Unit 7 (G1)	1.000	1.027	1.027	1.000	1.027	1.027
Unit 8 (G1)	1.000	1.000	1.000	1.000	1.000	1.000
Unit 9 (G1)	1.040	1.083	1.127	1.040	1.122	1.167
Unit 10 (G1)	1.000	0.996	0.996	1.000	0.992	0.992
Unit 11 (G1)	0.954	1.084	1.034	0.954	1.089	1.038
Unit 12 (G2)	1.000	0.949	0.949	1.000	0.939	0.939
Unit 13 (G2)	1.063	0.929	0.988	1.063	0.900	0.957
Unit 14 (G2)	1.008	1.164	1.173	1.008	1.096	1.105
Unit 15 (G2)	1.000	0.971	0.971	1.000	0.979	0.979
Unit 16 (G2)	1.063	1.082	1.150	1.063	1.082	1.150
Unit 17 (G2)	1.189	0.966	1.149	1.189	0.966	1.149
Unit 18 (G2)	1.058	1.022	1.082	1.058	1.022	1.082
Unit 19 (G2)	1.000	0.989	0.989	1.000	1.059	1.059
Unit 20 (G2)	0.987	0.966	0.953	0.987	0.966	0.953
Unit 21 (G2)	1.000	1.130	1.130	1.000	1.002	1.002
Unit 22 (G2)	1.190	1.074	1.279	1.190	1.074	1.279
Unit 23 (G2)	1.098	0.879	0.965	1.098	0.879	0.965
Unit 24 (G3)	1.000	1.136	1.136	1.000	1.035	1.035
Unit 25 (G3)	1.000	1.130	1.130	1.000	1.134	1.134
Unit 26 (G3)	1.000	0.958	0.958	1.000	0.998	0.998
Unit 27 (G3)	1.092	0.943	1.030	1.092	0.941	1.028
Unit 28 (G3)	1.249	0.935	1.168	1.249	0.956	1.194
Unit 29 (G3)	1.174	0.798	0.937	1.174	0.790	0.928
Unit 30 (G3)	1.237	0.784	0.970	1.237	0.784	0.970
Unit 31 (G3)	1.149	0.907	1.042	1.149	0.915	1.051
Unit 32 (G4)	1.000	1.115	1.115	1.000	1.000	1.000
Unit 33 (G4)	1.061	1.055	1.120	1.061	1.000	1.061
Unit 34 (G4)	1.374	0.733	1.007	1.374	0.733	1.007
Unit 35 (G4)	1.000	0.970	0.970	1.000	0.970	0.970
Unit 36 (G4)	0.986	0.848	0.836	0.986	0.890	0.877
Unit 37 (G4)	1.000	1.000	1.000	1.000	1.000	1.000
Unit 38 (G4)	1.053	1.039	1.094	1.053	1.028	1.082
Unit 39 (G4)	1.212	0.792	0.960	1.212	0.774	0.938
Unit 40 (G4)	1.079	0.950	1.026	1.079	1.000	1.079
Unit 41 (G4)	0.964	0.973	0.939	0.964	1.000	0.964

A further diagnosis of this drawback not only can provide specific reasons behind any change in the Malmquist index in the two approaches but also highlight other advantages of the proposed approach. For the determination of the Malmquist index of a maintenance unit *p*, it is required to compute the unit´s meta-efficiency against the frontier of the convex and non-convex meta-technologies in both time periods (2014 and 2015), i.e., $Eff^{M}(U_p^{2015})/Eff^{M}(U_p^{2014})$ in the convex form (see formula 4 in Section 2.1) and $Eff_{NC}^{M}(U_p^{2015})/Eff_{NC}^{M}(U_p^{2014})$ in the proposed non-convex form (see formula 13 in Section 2.3). These meta-efficiency scores $Eff^{M}(U_p^{2014})$, $Eff^{M}(U_p^{2015})$, $Eff_{NC}^{M}(U_p^{2014})$ and $Eff_{NC}^{M}(U_p^{2015})$ are reported in the second, fourth, sixth and eighth

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columns of Table 2, respectively. Moreover, the corresponding reference technologies involved in the computation of these efficiencies are also given next to the efficiency scores in this table.

Table 2. Meta-efficiencies and reference technologies of maintenance units in the two approaches

	Convex meta-frontier Malmquist index				*Non-convex meta-frontier Malmquist index*			
	Period 1		Period 2		Period 1		Period 2	
	Eff	**Ref. Technology**	**Eff**	**Ref. Technology**	**Eff**	**Ref. Technology**	**Eff**	**Ref. Technology**
Unit 1 (G1)	0.585	P1-G4, P2-G4	0.567	P1-G4	0.590	P1-G4	0.567	P1-G4
Unit 2 (G1)	0.555	P1-G4, P2-G4	0.586	P1-G4, P2-G4	0.559	P1-G4	0.589	P1-G4
Unit 3 (G1)	0.771	P1-G4, P2-G4	0.939	P1-G4, P2-G4	0.790	P1-G4	1.000	P2-G1
Unit 4 (G1)	0.849	P1-G4, P2-G2	1.000	P2-G1	0.929	P2-G2	1.000	P2-G1
Unit 5 (G1)	0.635	P1-G4	0.878	P1-G4, P2-G2	0.635	P1-G4	0.937	P2-G2
Unit 6 (G1)	0.616	P1-G4	0.696	P1-G4, P2-G2	0.616	P1-G4	0.711	P1-G4
Unit 7 (G1)	0.823	P1-G4	0.845	P1-G4	0.823	P1-G4	0.845	P1-G4
Unit 8 (G1)	0.950	P1-G4	0.950	P1-G4	0.950	P1-G4	0.950	P1-G4
Unit 9 (G1)	0.674	P1-G4	0.759	P1-G4, P2-G2	0.674	P1-G4	0.787	P1-G4
Unit 10 (G1)	0.635	P1-G4, P2-G4	0.632	P1-G4, P2-G4	0.648	P1-G4	0.642	P1-G4
Unit 11 (G1)	0.650	P1-G4, P2-G4	0.672	P1-G4, P2-G4	0.658	P1-G4	0.683	P1-G4
Unit 12 (G2)	0.741	P1-G4, P2-G4	0.703	P1-G4, P2-G4	0.792	P1-G4	0.744	P1-G4
Unit 13 (G2)	0.684	P1-G4, P2-G4	0.676	P1-G4, P2-G4	0.718	P1-G4	0.688	P1-G4
Unit 14 (G2)	0.775	P1-G4, P2-G4	0.909	P1-G4, P2-G4	0.884	P2-G4	0.976	P2-G4
Unit 15 (G2)	0.827	P1-G4, P2-G2, P2-G4	0.803	P1-G4, P2-G2	0.923	P2-G4	0.904	P2-G4
Unit 16 (G2)	0.529	P1-G4	0.609	P1-G4	0.529	P1-G4	0.609	P1-G4
Unit 17 (G2)	0.481	P1-G4	0.552	P1-G4	0.481	P1-G4	0.552	P1-G4
Unit 18 (G2)	0.563	P1-G4	0.609	P1-G4	0.563	P1-G4	0.609	P1-G4
Unit 19 (G2)	0.705	P1-G4, P2-G4	0.697	P1-G4, P2-G4	0.719	P2-G4	0.761	P1-G4
Unit 20 (G2)	0.768	P1-G4	0.732	P1-G4, P2-G4	0.768	P1-G4	0.732	P1-G4
Unit 21 (G2)	0.885	P1-G4, P2-G2	1.000	P2-G2	0.998	P1-G4	1.000	P2-G2
Unit 22 (G2)	0.640	P1-G4	0.818	P1-G4	0.640	P1-G4	0.818	P1-G4
Unit 23 (G2)	0.452	P1-G4	0.437	P1-G4	0.452	P1-G4	0.437	P1-G4
Unit 24 (G3)	0.673	P1-G4, P2-G2	0.764	P1-G4, P2-G2	0.748	P2-G1	0.774	P2-G2
Unit 25 (G3)	0.764	P1-G4	0.864	P1-G4, P2-G4	0.764	P1-G4	0.867	P1-G4
Unit 26 (G3)	0.730	P1-G4, P2-G4	0.699	P1-G4, P2-G4	0.749	P1-G4	0.747	P1-G4
Unit 27 (G3)	0.475	P1-G4, P2-G4	0.490	P1-G4	0.476	P1-G4	0.490	P1-G4
Unit 28 (G3)	0.515	P1-G4, P2-G4	0.601	P1-G4, P2-G4	0.515	P1-G4	0.615	P1-G4
Unit 29 (G3)	0.501	P1-G4, P2-G4	0.470	P1-G4	0.506	P1-G4	0.470	P1-G4
Unit 30 (G3)	0.536	P1-G4	0.520	P1-G4	0.536	P1-G4	0.520	P1-G4
Unit 31 (G3)	0.558	P1-G4, P2-G4	0.581	P1-G4, P2-G4	0.564	P1-G4	0.593	P1-G4
Unit 32 (G4)	0.897	P1-G4, P2-G2, P2-G4	1.000	P2-G4	1.000	P1-G4	1.000	P2-G4
Unit 33 (G4)	0.893	P1-G4, P2-G2	1.000	P2-G4	0.942	P1-G4	1.000	P2-G4
Unit 34 (G4)	0.541	P1-G4	0.545	P1-G4	0.541	P1-G4	0.545	P1-G4
Unit 35 (G4)	1.000	P1-G4	0.970	P1-G4, P2-G4	1.000	P1-G4	0.970	P1-G4
Unit 36 (G4)	1.000	P1-G4	0.836	P1-G4, P2-G4	1.000	P1-G4	0.877	P1-G4
Unit 37 (G4)	1.000	P1-G4	1.000	P2-G4	1.000	P1-G4	1.000	P2-G4
Unit 38 (G4)	0.484	P1-G4, P2-G4	0.529	P1-G4, P2-G4	0.512	P2-G4	0.554	P1-G4
Unit 39 (G4)	0.625	P1-G4, P2-G4	0.600	P1-G4	0.639	P1-G4	0.600	P1-G4
Unit 40 (G4)	0.829	P1-G4, P2-G4	0.851	P1-G4, P2-G4	0.927	P1-G4	1.000	P2-G4
Unit 41 (G4)	0.713	P1-G4, P2-G4	0.669	P1-G4, P2-G4	0.717	P1-G4	0.691	P2-G4

As can be seen in Table 2, the efficiency values determined by the convex meta-frontier Malmquist index are less than or equal the respective efficiencies computed by our approach. This derives from the fact that the non-convex PPS is a subset of the convex technology set (see Section 2.3). When different reference group technologies are applied, discrepancies between the efficiency scores arise. The same efficiency values, however, can be observed where the group technologies are identical under the convex and non-convex meta-frontier approaches.

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Let us take unit #22 as an example, which has received the same efficiency scores in both approaches (0.640 and 0.818 in 2014 and 2015, respectively). The reason is that the two frameworks have used the same reference group technology P1-G4 (i.e., group technology #4 in period 2014) as well as the same benchmarking peers for this unit (i.e., units #35 and #37). This can mainly be traced back to the size of this maintenance unit. Unit #22 is one of the smallest units in the whole data set in terms of both the THT and FTE values. Since the VRS-specification seeks for benchmarks of a similar size for this unit, only a few comparable maintenance units remain. This increases the probability that both approaches identify the same references, leading also to the same meta-efficiency scores. Investigating the results for this unit, we have found out that these identified references are also comparable regarding their other characteristics. For example, unit #22 and also its peers (units #35 and #37) have in common that they are located in big German cities, experiencing a similar business environment. Discussing this result with KONE, it has been confirmed that this precise way of selecting peers is recognized as a powerful feature of our approach by management.

As the results in Table 2 show, the convex meta-frontier Malmquist index often uses a combination of distinct group technologies from different time periods for the determination of the meta-efficiencies. For example, $Eff^{M}(U_{15}^{2014})$ is based on the reference technologies P1-G4, P2-G2 and P2-G4, i.e., the local technology of group #4 in the first and second period together with group #2 in the second period. However, evaluating units on the basis of such combinations of local technologies seems counter-intuitive from a practical point of view. The application of the convex meta-frontier Malmquist index explicitly accepts that all observations – regardless of their respective groups or time periods – can form the meta-technology. Therefore, not only for this unit, but also for all other units in different time periods, this framework does not distinguish between observations which are originated from different local technologies. In other words, observations influenced by a different internal and external environment have constructed together a meta-technology to measure the meta-efficiencies. According to the primary setting of the problem – which suggests grouping of units – and also the fact that the technology has changed over time (see again Section 3.2 for a few examples of reasons behind changes in the technology), we conclude that such combinations of the local technologies cannot be accepted as an accurate way of measuring efficiency.

By contrast, the proposed Malmquist index is immune to this problem. As can be observed in Table 2, our proposed approach uses, e.g., solely P2-G4 as a reference technology for measuring $Eff_{NC}^{M}(U_{15}^{2014})$. In other words, the non-convex approach does not make use of combinations of

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different technologies in the determination of the meta-efficiencies. This property not only leads to a more accurate estimate of the meta-technology but also preserves the characteristics of the local technologies in the form of the best experienced technology over time. This unique feature of the proposed approach plays a crucial role for managing the groups of maintenance units, where, e.g., improving their management styles and promoting corporate learning between groups are sought. For example, our approach identifies P2-G4 as an appropriate benchmark reference technology for unit #15 and some other units. This information can serve KONE as a starting point for a more detailed analysis of P2-G4. Intra-organizational learning seminars, e.g., can be used to analyze the special characteristics of this local group technology, which can be subsequently tested for their applicability to other maintenance units or groups in general and for unit #15 in particular. This example shows how the new approach can substantially support the corporate learning inside KONE and, hence, serve as an additional measure to gradually improve the productivity of the company.

4. Conclusions and outlook on future research

In this paper, we have proposed a new way of estimating the meta-technology, which only applies the minimum extrapolation principle on the aggregation of the experienced group technologies over time. We have shown theoretically as well as numerically that the resulting new meta-frontier Malmquist index, called the non-convex meta-frontier Malmquist index, provides more accurate results compared to the existing meta-frontier Malmquist index. The proposed index also preserves the role of each group technology – observed at a specific time period – in the estimation of the meta-technology. This includes information about the superior group technologies observed over time. As exemplified for the case of KONE, this unique feature of the suggested approach can play a crucial role in measuring and analyzing productivity, where a further diagnosis of individual performances is required. With respect to both computational and test properties, the proposed index also possesses the circularity property and is immune to infeasibility under VRS. Similar to traditional indices, it has been decomposed into the standard components such as efficiency change and best practice change.

From a practical point of view, future research will concentrate on analyzing the dual role of maintenance groups as both providers of services to the customers and operating units which should contribute to the profit of the KONE corporation as a whole. Therefore, the performance of KONE´s maintenance units should be assessed regarding financial objectives and compared with the operational efficiency measured in this paper. From a theoretical point of view, an

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interesting perspective for future research would be to extend the proposed non-convex meta-frontier approach to other DEA-based frameworks which implicitly accept convex combinations across distinct technologies either to measure the performance in a static setting or in a dynamic environment. For example, frequently applied approaches as the DEA window-analysis (e.g., Charnes et al. 1984) or the sequential Malmquist index (e.g., Shestalova 2003) use a structure which is quite similar to the conventional meta-frontier Malmquist index.

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